Forest Land Valuation in Washington State: Comparison of Abstraction and Regression

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In 1971 THE Washington State legislature revised the tax system used to assess privately owned forest land and the timber growing thereon. The new tax system replaced a locally administered annual ad valorem tax on timber with a centrally administered yield tax of 6 1/2 percent of the harvest value paid by the harvester. The State Department of Revenue (DOR) has the responsibility for administering the law.

Under the law, eligible forest land continues to be subject to an annual ad valorem tax based solely on the value of bare land. To be eligible, land must be dedicated to the growing and harvesting of timber. The DOR is responsible for the state-wide determination and annual certification of the true and fair value of each grade of forest land. Presently three site classes and four access-topography classes are used to grade each class of forest land.

The objectives of this paper are to: (a) discuss and critically review the valuation procedure used for determining the true and fair value of bare forest land in Washington State and (b) propose an alternative valuation procedure based on accepted statistical estimation procedures. The two procedures are applied to the mass appraisal of forest land in western Washington for both the 1975 and 1976 assessment years.

The Appraisal Process

There have been many attempts during the past two centuries at measuring the value of forest land (for example see Faustman[1], Fairchild[2], Williams[3], and Judeich[4]). These past attempts have all revolved around one or more of the three traditional approaches to valuation: the market, income, and cost approaches. For a variety of reasons the market approach has been the most often used method in the assessment of real property (Williams and Canham,[5] p. 6). As pointed out by Sizemore et al.[6] (pp. 25–26), the courts usually favor the use of market evidence when passing judgment in real property valuation cases. Further, economists generally favor the market approach, asserting that the price preference mechanism reveals the "true" marginal value of a unit area. Assuming a perfect market (i.e., perfect information, product homogeneity, and large numbers of buyers and sellers under no compulsion to buy or sell and with no market domination) transactions reveal the real value of the property being traded at a particular time. The market approach remains the most common approach to assessment even in the less-than-perfect real estate and forest land markets.

The Washington State DOR has relied on the market approach for determining bare forest land values since

1. Faustman, M. 1849. Calculation of the value which forest land and immature stands possess for forestry. Reproduced in Institute Paper No. 42, Commonwealth Forestry Institute, University of Oxford, 1968. Translated by W. Linnard.

2. Fairchild, F. R. 1935. Forest taxation in the United States. U.S. Department of Agriculture Misc. Publ. 218, 681 p.

3. Williams, E. 1968. Progress in the assessment of forest land and timber, 1956–66. *Assessors Journal* Vol. 2, p. 25–35.

4. Judeich, F. 1965. The Austrian Cameral valuation method. Forestry Chronicle, Vol. 41, No. 1, p. 84–92. Translated by T. W. Dwight and P. Jaciw, originally written in 1788.

5. Williams, E., and H. Canham. 1972. The productivity concept in forest taxation. *Forest Science* Vol. 18, No. 1, p. 3–20.

6. Sizemore, W., M. Sizemore, A. Herrick, and L. Hargreaves. 1965. Estimating fair market value of southern forest lands. Georgia Forest Research Council Report No. 15, University of Georgia, Athens, Georgia. 120 p.

mates of bare land value as 8 of the 14 equations included bare acres. Specifically, these per-acre estimates were \$115, \$119, \$510, \$68, \$108, −\$210, \$46, and −\$174. The simple average of these is \$72.75; the weighted average is \$71.42; the simple average of the positive values is \$161; and the weighted average of the positive values is \$106.36. None of these values was selected. Instead \$104.36 was used: the weighted average after discarding the highest (\$510) and the two lowest estimates. The sales contributing to these estimates were not discarded; only the information concerning the bare acres was ignored. In the second iteration, the following per-acre bare land values were obtained: \$104, \$167, \$87, \$121, \$119, \$115, \$445, and \$65. This time the appraiser discarded the five highest estimates involving 57 percent of the bare land acres. The weighted average of the three lowest estimates retained was computed as \$93 per acre. This estimate was subsequently used in the third iteration. A similar process was followed through five iterations.

In addition to the sequential estimation of bare land values, the abstraction process estimates the value of three classes of immature timber—conifer reproduction, conifer immature, and hardwood.[10] Thus, at each iteration a total of four value estimates are produced. Again, selected pieces of information are discarded throughout the estimation process. The process of sequential estimation is terminated when a set of "reasonable" relationships between the four value elements are obtained and when the estimated and actual total sales prices are approximately equal.

In the 1975 western Washington abstraction the fourth iteration produced the following per-acre estimates:

(a) bare land: \$88,

(b) conifer reproduction and land: \$211,

(c) conifer immature and land: \$317, and

(d) hardwood reproduction, immature, and land: \$161. These estimates produced a +2 percent deviation over the actual total sales price for the 84 valid sales. Thus, in the fifth, and final, iteration the entire downward adjustment was allocated to bare land while leaving the other estimates unchanged.[11] Substituting the final estimates into equation (2), the predicted total sales price, $\Sigma \hat{y}_i$, is

$$\Sigma\ \hat{y}_i = \$77(6004) + \$211(7338) + \$317(1684) + \$161(4133) = \$3{,}209{,}867$$

The per-acre dollar values represent (in order) bare land, conifer reproduction and land, conifer immature and land, and hardwood reproduction, immature and land, respectively. The parenthetical figures indicate the number of acres in each value class. The actual total sales price for the 1975 western Washington sales data was \$3,207,290. Thus, a 0.08 percent deviation is achieved when the estimated total is expressed as a percent of the actual total sales price.

As stated earlier, an infinite set of estimates exists which will produce a similar zero percent deviation. For example, consider the following examples derived from the 1975 data base:

1. \$100(6004) + \$196(7338) + \$301(1684) + \$160(4133) = \$3,206,812 (−0.015 percent deviation)
2. \$104(6004) + \$203(7338) + \$281(1684) + \$150(4133) = \$3,207,184(−0.003 percent deviation)
3. \$90(6004) + \$185(7338) + \$279(1684) + \$203(4133) = \$3,206,725(−0.018 percent deviation)

Each of these sets of estimates pass the minimum difference criterion and produce results which could be judged "reasonable." Yet, using abstraction there is no objective way to distinguish between the estimates. Instead, the bias of the appraiser is allowed to play a significant role in the estimation process. While judgment always plays a significant role in property appraisal, extra care and caution must be exercised in a mass appraisal when an inference about value is based on a small number of actual transactions.

In any mass appraisal the need for an objective unbiased procedure should be of paramount concern to the appraiser. It is clear that abstraction cannot produce estimates free from the appraiser's bias. Furthermore, the process presents a facade of objectivity. In fact, as used in Washington the subjective opinion of the appraiser and not valid sales evidence forms the basis of the process. While the judgment and experience of the appraiser are important components of the appraisal process, they must not be confused with, or used as substitutes for, sound, objective valuation methodologies. This is especially critical in a mass appraisal application.

Multiple Regression Analysis

Return to the point in the appraisal process where the collection, validation, and field checking of all valid forest land sales have been completed. To this point the appraiser has used his experience and judgment in screening sales for inclusion or exclusion in the data base. However, once this process is completed, the appraiser's task becomes one of value estimation. To the extent that the carefully screened sales reflect the forest land market, they contain information concerning land and immature timber values which, when properly estimated, will be applied to all untransacted properties. In such a mass appraisal, the appraiser is faced with an almost classical application of statistical

10. Hardwood reproduction and immature timber are considered in the aggregate.

11. This procedure was changed in 1976 for western Washington when each value estimate was adjusted proportionally on the basis of the number of acres in each value class.

estimation.[12] As such the value estimation procedure must conform to certain rules of scientific objectivity.

Once the sales data have been collected, the appraiser's job is to establish a relationship between selected value elements which characterize the parcels of forest land included within the data base. For example, he may wish to predict the average net sales price as a function of the number of acres involved in the sale. Accordingly, he might postulate that the relationship is

$$y_i = \beta x_i + \epsilon_i \quad (3)$$

where

y_i = The average or expected net sales price given some value of x_i – the number of acres in the sale area.

β = A coefficient which measures the slope of the assumed linear relationship between x_i and y_i. In the context of this problem β measures the per-acre dollar value of land.

ϵ_i = The error or deviation of the actual net sales price from its average or expected value.

Assuming that the above postulated functional relationship, or model, is correct, the next task is to derive an estimate of β. Examples of common estimation techniques are the method of moments, the method of maximum likelihood, Bayesian estimation, ratio estimation, and regression estimation using the method of least squares. Since each of these procedures generally yields a different estimate when applied to a particular set of data, it is necessary to know which estimator will yield the "best" estimate under stated conditions. Various statistical properties of estimators are used for this purpose. Among these properties are unbiasedness, minimum variance, relative efficiency, consistency, and sufficiency. Because the problem we are considering involves the estimation of parameters of a linear functional relationship (see equations 1 and 3) the following discussion is limited to regression estimation using least squares.

Bruce and Sundell[13] have illustrated the extent to which multiple regression analysis (MRA) has been accepted by the appraisal profession with their review of the literature covering approximately 100 articles. Shenkel[14] in discussing the use of MRA concludes that it is particularly adapted to income producing properties. Thus, there is little reason to doubt the assertion that "MRA has proved beyond a doubt its ability to improve assessment performance in a favorable environment."[15]

Utilizing the same data collected for abstraction, we now turn to a discussion of MRA and its application to the 1975 and 1976 western Washington forest land sales data. The postulated model; used throughout the MRA was previously described by equation (1). This is the same model underlying abstraction. It is linear in the unknown coefficients (θ_i). As in abstraction, the intercept (constant) is set equal to zero, implying that a sale of zero acres results in a zero sales price. Given that the model is correctly specified,[16] the least squares regression estimates of the θ_i in equation (1) are statistically unbiased. As shown by Bare (1975) the error terms (ϵ_i) in equation (1) for the 1975 western Washignton sales data are normally distributed with a mean of zero. However, the variance of the error was found to increase with increasing parcel size. In order to obtain the best linear unbiased estimate of each θ_i, it is necessary therefore to use weighted least squares regression analysis. A similar result was obtained for the 1976 western Washington data.

Using the 1975 and 1976 western Washington sales data, a plot of error terms from the unweighted least squares fit of equation (1) suggested that the variance of the error term was increasing in proportion to the squared value of parcel size. As shown by Bare[17], a weight equal to the squared inverse of parcel size produced a stabilized constant variance.[18] With this modification for unequal variance the actual model was altered to the following form:

$$Y_i/Ac = \theta_1 X_{i1}/AC + \theta_2 X_{i2}/AC + \theta_3 X_{i3}/AC + \theta_4 X_{i4}/AC + \epsilon_i \quad (4)$$

where Y_i is the DOR's trended net sales price and AC is total parcel size. Under the stated assumptions, weighted least squares regression produces the best (i.e., minimum variance) linear unbiased estimate of the θ_i. Because of the normality of the error term, it is also possible to make valid tests of hypotheses and to establish confidence intervals about the estimates.

The weighted least squares regression estimates for the 1975 and 1976 assessment years for the θ_i in equation (4) are shown in Table 1. As a measure of the goodness of fit of the model, the ratios of the sum of

12. Statistical estimation is the process of making an inference from a representative sample to a target population in an objective manner which permits one to express the confidence in the validity of the estimate.

13. Bruce, R. W., and D. J. Sundell. 1976. Multiple regression analysis: History and application in the appraisal profession. *The Real Estate Appraiser*.

14. Shenkel, W. 1974. The valuation of income property by multiple regression techniques. *In* The Application of Multiple Regression Analysis in Assessment Administration. Proceedings of a symposium conducted by the International Association of Assessing Officers, Research and Technical Services Department in cooperation with the John C. Lincoln Institute. Published by the International Association of Assessing Officers, Chicago, Illinois.

15. Gloudemans, R. J. 1974. Introduction to the application of multiple regression analysis in assessment administration. Proceedings of a symposium conducted by the International Association of Assessing Officers, Research and Technical Services Department in cooperation with the John C. Lincoln Institute. Published by the International Association of Assessing Officers, Chicago, Illinois.

16. A detailed description of model development is under preparation.

17. Bare, B. B. 1975. An evaluation of the abstraction process of forest land valuation in the state of Washington. Unpublished mimeo, College of Forest Resources, University of Washington, Seattle. 35 p.

18. This weighting factor is equivalent to a transformation of the data obtained by dividing both sides of equation (1) by parcel size.

squares due to regression to the total sum of squares for the 1975 and 1976 assessment years were computed as 0.83 and 0.82, respectively. In Table 2 are the comparable abstraction estimates. Note that MRA has produced a more consistent set of value relationship over the two years than has abstraction. In particular, note that the abstraction value for conifer reproduction decreased $44 per acre between 1975 and 1976 while at the same time the abstraction value for conifer immature increased $17 per acre. As shown by Bare,[19] this is an example of the inconsistency of the value relationships that abstraction can produce. The regression estimates for conifer reproduction and conifer immature increased $20 and $16 per acre, respectively, between 1975 and 1976. This indicates that "reasonable" value relationships are consistently produced by the MRA.

In order to place much confidence in our interpreation of each regression coefficient, a check must be made to determine if multicollinearity is a problem. Reinmuth[20] and Skaff[21] provide excellent summaries of the effects of multicollinearity on individual coefficient interpretation. Farrar and Glauber[22] suggest several means of testing for the presence of multicollinearity in a data set. Following their procedures the determinant of the correlation matrix was computed for the 1975 and 1976 assessment years obtaining 0.94 for both years. At the 95 percent probability level we can't reject the hypothesis that the determinant equals one and hence we conclude that multicollinearity is not a significant problem. A visual inspection of the diagonal elements of the inverse of the correlation matrix also confirms this conclusion, as all coefficients are nearly equal to one. Furthermore, each regression coefficient is highly significant.

The existence of interactions among the set of inde-

19. Bare, B. B. 1976. Comparison of western Washington sales data used in abstraction and regression analyses for 1975 and 1976. *In* Department of Revenue Technical Subcommittee Report on Use of Multiple Regression Analysis in Forest Land Valuation, June 28, 1976, Olympia, WA., p. 111.

20. Reinmuth, J. 1974. The use of multivariate statistical methods in assessment analysis, with special emphasis on the problem of multicollinearity. *In* The Application of Multiple Regression Analysis in Assessment Administration. Proceedings of a symposium conducted by the International Association of Assessing Officers. Research and Technical Services Department in cooperation with the John C. Lincoln Institute. Published by the International Association of Assessing Officers, Chicago, Illinois.

21. Skaff, M. 1974. Implications of multicollinearity and interactive effects on the predictive ability of a mass appraisal model. *In* The Application of Multiple Regression Analysis in Assessment Administration. Proceedings of a symposium conducted by the International Association of Assessing Officers, Research and Technical Services Department in cooperation with the John C. Lincoln Institute. Published by the International Association of Assessing Officers, Chicago, Illinois.

22. Farrar, D. E., and R. R. Glauber. 1967. Multicollinearity in regression analysis: The problem revisited. ***Review of Economics and Statistics*** Vol. XLIX, No. 1, p. 92–107.

Table 1

Regression Estimates of Each Value Element
For Western Washington

	Assessment Year 1975 (n = 84) ($/acre)		Assessment Year 1976 (n = 117) ($/acre)	
	Estimate	Standard error[e]	Estimate	Standard error
θ_1[a]	128.17	16.08	144.93	14.69
θ_2[b]	191.80	18.26	212.36	18.20
θ_3[c]	271.44	35.47	287.05	35.50
θ_4[d]	188.88	21.99	190.30	19.46

[a] Per acre value of bare land.
[b] Per acre value of conifer reproduction and land.
[c] Per acre value of conifer immature and land.
[d] Per acre value of hardwood reproduction, immature and land.
[e] The standard error is the standard deviation of the regression coefficient.

Table 2

Abstraction Estimates of Each Value Element
For Western Washington

	1975 Assessment Year Estimate ($/acre)	1976 Assessment Year Estimate ($/acre)
θ_1[a]	77	83
θ_2	211	167
θ_3	317	334
θ_4	161	244[b]

[a] See Table 1 for definition of each θ.
[b] Weighted average of $138 per acre for hardwood reproduction and $201 per acre for hardwood immature.

Table 3

Comparison of Adjusted Bare Land Values
For an Average Acre in Western Washington

Assessment year	Unadjusted bare land value		Adjusted bare land value		Percent difference in adjusted values
	Abstraction ($/acre)	Regression ($/acre)	Abstraction ($/acre)	Regression ($/acre)	
1975	77	128.17	58.82	102.31	73.9
1976	83	144.93	64.59	118.43	83.4

pendent variables must also be addressed prior to acceptance of the regression estimates as per-acre values. Theoretically the per-acre value of forest land (i.e., the capital value of all incomes and expenses which occur until infinity) is equal to the sum of the land and stand values.[23] However, to test this theory the residuals from equation (4) were plotted against the trended net sales price. This plot revealed that the residuals were distributed about a mean of zero with constant variance. If any interaction terms were omitted, it is likely that this plot would have revealed a nonlinear trend. Since it didn't we conclude that each regression coefficient can be safely interpreted as a per-acre value.

Discussion of Results

As previously stated, all valid forest land sales, regardless of land grade, are treated in the aggregate in both the abstraction and regression analyses. Adjustments for site class, access and topography are made ex post facto. Attempts to post stratify the data base by site class, size of sale, and accessibility-topography class were not successful due to the small number of sales included in the data base. Therefore, all previously reported results are for the average acre of forest land represented by the western Washington forest land sales data.

Before applying the results of either abstraction or regression to the untransacted properties in western Washington, an adjustment from the average land grade in the data base to the assumed average land grade in western Washington must be made. Since the DOR is primarily concerned with bare land value, only the value estimates of bare land are adjusted. For both 1975 and 1976, the sales were determined to have above-average site, accessibility, and topography as compared to the assumed average of same. Therefore, downward adjustments in the value estimates were required. A comparison of the adjusted abstraction and regression results is shown in Table 3. As shown, abstraction under valued forest land in western Washington by 74 and 83 percent in 1975 and 1976, respectively, as compared with MRA. This is obviously a significant difference.

These results illustrate the magnitude of the differences in assessed value using the two approaches. When faced with such a discrepancy, the common approach is to reexamine the underlying philosophy and procedures embodied in each valuation methodology. The final selection should favor that methodology which is based on the most logical, objective, and consistent set of premises.

Both abstraction and MRA employ the same basic model (i.e., eq. 1) and use the same sales evidence. Thus, a deficiency in either input equally affects both valuation procedures. Common problems are model mis-specification or data that are not characteristic of the population being sampled. Neither of the above are in contention here as all available evidence to date suggests that the basic model is correctly specified and that the valid forest land sales included in the data base are representative of the untransacted properties in western Washington.

We conclude, therefore, that differences in the assessed value of bare forest land as shown in Table 3 are due to the valuation procedures themselves. The magnitude and direction of differences are specific to this case. The MRA regression estimate is determined by relationships inherent in the data. Over- or undervaluation by abstraction relative to the regression estimate of true market value is a function of the individual appraiser who applies abstraction.

In summary the major weaknesses of abstraction are:

(a) an arbitrary set of value estimates can be derived which pass the minimum difference criterion (i.e., zero deviations between actual and expected net sales price) and which bear a "reasonable" relationship to each other, and

(b) judgment and intuition may be freely substituted for the objective and statistical analysis of market evidence. MRA is preferred to abstraction because it is a well-established statistical estimation procedure which produces a unique best fit. Therefore, we feel that MRA should replace abstraction for the mass appraisal of forest land in western Washington.

23. Faustman, op. cit.

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